

Available online at www.sciencedirect.com



Journal of Sound and Vibration 278 (2004) 1163-1179

JOURNAL OF SOUND AND VIBRATION

www.elsevier.com/locate/jsvi

Evolution of coherent structures and feedback mechanism of the plane jet impinging on a small cylinder

Fei-Bin Hsiao^{a,*}, I-Che Hsu^a, Jiann-Min Huang^b

^a Institute of Aeronautics and Astronautics, National Cheng Kung University, Tainan 70101, Taiwan, ROC ^b Aeronautical Systems Research Division, Chung Shan Institute of Science and Technology, Taichung, Taiwan, ROC Received 28 January 2003; accepted 24 October 2003

Abstract

The dynamics of coherent structures and their instability evolution of the small cylinder impinging plane jet are extensively studied in this paper. The hot-wire measurements are conducted to investigate the physical flow properties and relevant flow field behaviors. The jet exit velocity is operated between 2-26 m/s, with the Reynolds number range from 2.1×10^3 to 2.7×10^4 based on the initial jet width. The acoustic excitation is applied to verify the instability evolution mechanism and is introduced at the jet nozzle exit with two arrays of small loudspeakers. A modified feedback mechanism is proposed to successfully explain the interaction between the cylinder and the plane jet. The competition between the jet and wake shear layer instabilities is significantly revealed in comparison with the standing wave number measured in the self-sustained oscillating flow. The augmentation effect of acoustics excited at the resonant frequency on the impinging flow with the small cylinder is also explored in detail to substantiate the feedback mechanism.

© 2003 Elsevier Ltd. All rights reserved.

1. Introduction

For most of the impinging jet flow investigations reviewed by Rockwell and Naudascher [1] before, the obstacle structures in size for impingement are normally much larger than the jet exit height. However, the small cylinder impinging plane jet has been another research topic of interest pertaining to the study of the two-dimensional impinging jet flow where a circular cylinder having a relatively smaller diameter, d, compared to the plane jet exit height, H, of $d/H \leq 0.4$, is located in the potential core region of the plane jet. Chou et al. [2] and Hsiao et al. [3] further investigated the small cylinder impinging plane jet flow, where the jet and the small cylinder wake coexist and

^{*}Corresponding author. Tel.: +886-6-275-7575 ext. 63667; fax: +886-6-238-9940. *E-mail address:* fbhsiao@mail.ncku.edu.tw (F.-B. Hsiao).

interact with each other to generate very intriguing flow phenomena in the impinging flow field study.

In fluid mechanics, jet, wake and mixing layer are of three basic types in free shear flows. In fact, a large number of researches on turbulent shear flows have been carried out for several decades. This is simply because they are genetically of importance in many practical applications, such as fuel-air mixing in combustors, jet ejectors, noise reduction and high lift devices, etc. For jets, Crow and Champagne [4] and Brown and Roshko [5] first confirmed that coherent structures stem the intrinsic features of jet flow development. Winant and Browand [6] subsequently showed that the growth of jet is governed by the phenomenon of vortex formation and merging processes. Later, Ho and Nosseir [7] and Ho and Hsiao [8] proposed the subharmonic evolution model, which interprets that vortex formation and merging processes are due to the results of the evolution of fundamental and subharmonic instabilities. With the introduction of acoustic excitation on jet, Sato [9] found that different initial modes evolution could lead to distinct growth rates of the shear layers at specific instability frequencies. Ho and Gutmark [10] further confirmed that the free jet can be effectively excited by acoustic waves in its inherent frequency to obtain more upstream vortex saturation position and more localized, organized and higher energy-contained coherent structures.

For the wake flow behind a two-dimensional circular cylinder, Strouhal [11] first found the relationship of the Aeolian tones and the wind speed. Von Karman [12] further analyzed in detail the stability of vortex street configurations and established the relationship between the drag and the vortex street. From then on, numerous researchers have deeply devoted to investigate the dynamical behaviors of the Karman vortex street in wakes behind various geometries of obstacles. Williamson [13] recently reviewed and summarized a list of works regarding the vortex dynamics in the cylinder wake, showing the intrinsic beauty of these studies.

Another relevant topic of flow research is the impinging shear flow. For those mentioned by Rockwell and Naudascher [1] are such as jet-edge, jet-slot, jet-cylinder, jet-ring, jet-hole, jet-plate, jet-flap associated with axisymmetric jets, mixing layer-edge, rectangular cavity, axisymmetric cavity, and special cavity associated with mixing layers. The most gripping part of this kind of impinging flows is concerning the interaction between the coherent structures and the impinged body. The flow in the interacting region is also called the self-sustained oscillating flow, which comes from the governing mechanism due to the impingement effect. Ho and Nosseir [7] examined the axisymmetric jet impinging on a flat plate and found the phase locked phenomenon. They suggested that a feedback loop is created by the propagating pressure waves induced from the coherent structures that impinge on the solid body located downstream. Rockwell and Naudascher [1] explored a series of problems related to the wedge and corner impingement via jet and mixing layers. They found that the flow impingement of the coherent structures upon the body indeed resulted in upstream influence and further amplify the evolving instabilities within the shear layers.

This paper is further to study the evolution of coherent structures and their feedback mechanism in the small cylinder impinging plane jet, in particular with the introduction of acoustic waves as an excitation in producing velocity perturbations at the initial stage of the jet development. Detailed investigations in the present study include the velocity flow field development, the competition between the jet and wake shear layer instabilities, the standing waves in the self-sustained oscillating flow phenomena,

and the feedback mechanism for the small cylinder impinging plane jet flow. The hot-wire measurements are employed in the study. The cylinder with the diameter of 3 mm is used as an obstacle for impingement, which is relatively small in comparison with the jet nozzle height of 15 mm. The impinging jet is locally excited at the nozzle exit of the jet flow by two arrays of small loudspeakers at various frequencies.

2. Experimental apparatus and data processing

The experiments were carried out in a plane jet facility with a height (*H*) of 15 mm and an aspect ratio of 20 at the nozzle exit. The jet nozzle is a fifth order polynomial profile with the area contraction ratio of 20. For the operating velocity of 10 m/s, the turbulence intensity at the nozzle exit center is less than 0.2%. The Reynolds number based on the jet exit height ranges from 2.1×10^3 and 2.7×10^4 , with the operating velocity (U_0) from 2 to 26 m/s. A small steel cylinder of 3 mm in diameter is rigidly and carefully placed across the whole span of the plane jet, which is movable by a traverse mechanism. The steel cylinder is securely fastened with tension to ensure there will be no natural oscillations due to the jet impingement.

In order to perform very low-level acoustic pressure disturbance excitation with the sine wave signals, 20 earphone-type loudspeakers are used as excitation sources, which are placed at both sides of the nozzle exit. They are evenly distributed and are glued on two aluminum beams, which are mounted near the lips of the nozzle with a 5 mm offset from the nozzle edges without creating any flow interference (see Fig. 1). In addition, each beam has a small slit of 2 mm in width facing to the jet flow. Each earphone is wired in parallel with its own linear amplifier in order to produce identical magnitude and phase of sound for performing acoustic excitation.

The velocity measurements are conducted by the lab-made hot-wire anemometers using a cross hot-wire probe with 5 μ m Pt wire in diameter as the sensing element. A 16-bit A/D converter with sample-and-hold capability is connected to a personal computer (PC) for data acquisition. The Cartesian co-ordinate system is used in the study, as shown in Fig. 1 with the origin located at the middle height of the nozzle exit. The probe and cylinder movements are done through a



Fig. 1. Schematic representation of the acoustic excitation apparatus and the flow field co-ordinate.



Fig. 2. Block diagram of the phase-averaged technique.

two-dimensional traversing mechanism. Throughout the measurements, the probe is set at the middle of the jet height to avoid the three dimensional effect introduced by the nozzle edges.

In the vortex dynamics studies, the conditional phase-averaged processing technique is employed and sketched in detail in Fig. 2. One reference probe is placed in the flow field where strong and repeating signals can be detected. To accurately determine the initial point of a wanted reference signal, the instantaneous transverse velocity is first passed through a low pass filter to filter out either the unwanted noise or the higher frequency part. After sliced by the predetermined reference signal, the statistic estimation is applied. The initial phase with the largest probability is chosen to stand for the flow properties at this measuring point. The throughput at this stage excluding due to the phase jittering phenomenon in jet shear layer may range from 60% to 80%. Finally, the raw data of the instantaneous velocities are retrieved and segmented again. After all, the number of data set for ensemble averaging is at least 150 for all conditional phase average processes.

The experimental errors in the present study can be categorized into position error and calibration error. The measuring volume of the X-type probe is $2 \text{ mm} \times 2 \text{ mm}$. The transverse origin of the hot-wire probe and the cylinder is aligned to which the resulting mean velocities distribution show symmetry. According to the measuring resolution, the position uncertainty of the hot-wire probe and the cylinder in transverse direction based on the nozzle height is less than 1%. The position error in streamwise direction is less than 0.2% of the nozzle height. The pressure transducer was calibrated by a micro-manometer with the accuracy of $\pm 0.005 \text{ mmH}_2O$. The total errors induced from the repeatability and the calibration inaccuracy are about 2%. Throughout the investigations, the spectrum analysis is accomplished by the ensemble averaging of 16 data sets with 4096 sample points each and acquired at 4 K Hz sampling rate.

3. Results and discussion

3.1. Free jet and wake

The characteristics of the laminar plane jet are first examined to really exhibit the top hatted profile as well as the Tanh-profile of the mean streamwise velocity distributions at the jet nozzle

exit. Linear growth pattern of the momentum thickness and volume entrainment developments are also checked. The evolution of the coherent structure shows three stages at the dominant passage frequency distributions (i.e., fundamental and the subharmonic instabilities) in streamwise direction. As shown in Fig. 3, the dominant frequency along the inner shear layer at $U = 0.9 U_0$, which is called the passage frequency. It exhibits three stages pattern, which respectively represents the fundamental, first subharmonic and second subharmonic of the evolving instabilities. The relative amplitudes of the dominant instabilities also behave such features as growth, saturation and attenuation in the conventional shear flow dynamics. These are qualitatively identical to the well-known laminar plane jet properties and quantitatively agreeable with our previous study (Hsiao and Huang [14]). Furthermore, the eigeninstability frequency, f_0 , of the laminar plane jet varies with the jet exit velocity U_0 at the power law of 1.5, as shown in Fig. 4. The resulting nondimensional frequency, the so-called Strouhal number St_0 defined as $f_0\theta_0/U_0$, is 0.017, which lies within the reasonable extent proposed by Ho and Gutmark [10]. On the other hand, the shedding vortices in the cylinder wake has an eigeninstability frequency known as the shedding frequency, which is denoted as f_w here. Its non-dimensional Strouhal number St_w , which is $f_w D/U_0$, is measured to be 0.18. The relation between f_w and U_0 (U_0) represents the freestream velocity of the wake here) is also depicted in Fig. 4 for comparison, where the power law of one is clearly demonstrated. It is noted that this is in contrast with the result for the laminar jet flow whose power law of instability frequency with the jet velocity is 1.5.

As for the mode shape development of the instabilities, there exist two instability modes in the plane jet shear layers, which are the varicose mode (symmetric) and the sinuous mode (antisymmetric). From the flow visualization results by Huang and Hsiao [15], the mode shape can be directly distinguished from the vortex arrangements located in the two shear layer regions, where the two steams of vortices may exhibit symmetric (varicose) or staggered (sinuous) with respect to the jet centerline. By employing two hot-wire probes located in each side of the jet shear



Fig. 3. Dominant passage frequency along the stream direction at $U_0 = 10$ m/s.



Fig. 4. Instability frequencies of jet and wake with respect to freestream velocities.

layers, one can obtain either symmetric (varicose, m = 0) or antisymmetric (sinuous, m = 1) patterns from the streamwise velocity fluctuations u(t). However, concerning the transverse velocity fluctuation v(t), its phenomena are clearly reversed due to flow conformity in the continuity equation. In particular, the calculations of the correlation function coefficient R_{12} of either streamwise or transverse velocity fluctuations at the time delay equals to zero clearly demonstrate the features of the mode shape development. The detailed discussion about the flow structure development can be referred to our previous work (Hsiao and Huang [15]). With regard to the cylinder wake flow, the mode shape development of the shedding instability is very much identical to the jet flow and can directly be determined from the fluctuating velocity measurements.

3.2. Feedback mechanism of a self-sustained oscillation flow

Concerning a flow system having existence of resonance instability at some specific frequencies, Ho and Nosseir [7] proposed the feedback mechanism to govern the dynamics of the self-sustained oscillating flow. Fig. 5 shows how this feedback mechanism works within the impinging region. The instability wave travels downstream in appearance of coherent structures with the convective speed (U_c). As the coherent structures hit on the surface of the impinging body, the generated pressure wave is propagated upstream with the sound speed (a) of the medium where it travels. It is like two standing waves interacting with each other and travelling back and forth between two solid walls shown in Fig. 5. Therefore, this will be a resonance case with the same resonant



Fig. 5. The feedback mechanism proposed by Ho and Nosseir [7].

frequency f_r , and the total number of waves of these two waves should be equal to an integer N. It is known that the number of waves (N) can be calculated from the travelling distance L, which is also called the impinging length, divided by the wavelength λ_r . The wavelength is the wave speed divided by the frequency f of the wave. Since the experimental freestream velocity U_0 in the present study is in the subsonic range, the sound speed a is then much larger than the convective speed U_c of the flow. The number of waves then can be determined by a simplified equation as follows:

$$N = f_r \left(\frac{L}{U_c} + \frac{L}{a}\right) \cong \frac{f_r L}{U_c} = \frac{f_r L}{KU_0}.$$
(1)

Here, $K = U_c/U_0$ is assumed. Eq. (1) can also be written as

$$N \cong \frac{L}{\lambda_r} = \frac{f_r L}{U_c}.$$
(2)

This simplified feedback equation, either Eq. (1) or (2), is basically valid for the subsonic jet flow impingement and can be applied to the free shear flows with instability frequency where the vortex paring occurs or the flow impingement takes place.

3.3. The small cylinder impinging plane jet

In the present study, we focus on the impinging flow where a small cylinder is placed right in the center of a jet flow, thus creating a complex but interesting self-sustained oscillating flow with violent interaction between the free shear layers and the embedded wake vortices inside the jet flow. Following our previous studies in the impinging jet flow (Hsiao et al. [3,16]) and Hsu [17], the induced behavior of the dominant instability frequencies was first examined. As expected, the original fundamental frequency f_0 in a free jet is shifted to the resonance frequency f_r , which is near the wake shedding frequency f_w though. In the jet impinging flow, the upstream propagating disturbances are in the form of pressure wave from the cylinder surface generated by the shedding vortices. The initial most amplified instability is so excited under the shedding vortex induced disturbances, and then governs the downstream evolution of the coherent structures associated with the self-sustained flow oscillation. Furthermore, the resonance frequency f_r was found to vary with the jet exit velocity U_0 and the impinging length of the cylinder location L. In the meantime, the convective velocity U_c is about $0.42U_0$ in the jet shear layer region, while it is about $0.72 U_0$ in the wake shear layer region, indicating K is truly a constant value depending on the area of shear layers. Hsiao et al. [3] measured the dominant frequencies in the jet shear layer regions for various jet exit velocities and various cylinder positions along downstream. The results showed

that the variations of the resonant frequencies or the so-called frequency jumps are significant when the small cylinder moves along the streamwise direction of the jet center. Note that this is the case where the cylinder is small enough to be totally emerged in the potential core region of the jet flow. It seems that no coherent structures of the jet can impinge on the cylinder surface. As the cylinder positions are shifted farther downstream, while being kept on the jet centerline, an interesting self-sustained oscillating frequency jump is created due to the effect of the obstacle impingement to create the flow resonance due to feedback mechanism as discussed previously in Eqs. (1) or (2). The induced resonance frequency f_r is then mainly determined by the downstream position of the cylinder (i.e., the impinging length L), the jet exit velocity and the value K. However, the relative cylinder size does play an important role in the occurrence of the resonance frequency, and the overall consequence of resonance follows the feedback mechanism of Eq. (1). Fig. 6 shows the resonance frequency stage changes of the jet flow interaction with the small cylinder where the cylinder position varies in downstream distance with the Reynolds numbers tested. The verification of the standing wave in the self-sustained oscillating flow was accomplished by analyzing the phase component in the cross-correlation function, following the method by Hsiao et al. [3]. The results are illustrated in Table 1, where the number of waves (N) were found, such as 3, 5, 7, and 9 for $U_0 = 10$ m/s with the corresponding cylinder position L. Therefore, the instability dominance of the small cylinder impinging plane jet can be simplified



Fig. 6. Cylinder position corresponding to the frequency jump for each stage at various Reynolds numbers for $U_0 = 10 \text{ m/s}$.

Table 1

Calculations of frequency jumps with cylinder position according to the feedback equation Eq. (1) for $U_0 = 10 \text{ m/s}$

L(i + 1)/L(i)	N(i+1)/N(i)			
	L1 = 18.0 mm (1st jump)	L2 = 28.0 mm (2nd jump)	L3 = 39.0 mm (3rd jump)	L4 = 52.0 mm (4th jump)
L1 = 18.0 mm (1st jump)	N1 = 3	1.7	2.3	3
L2 = 28.0 mm (2nd jump)	1.6	N2 = 5	1.4	1.8
L3 = 39.0 mm (3 rd jump)	2.2	1.4	N3 = 7	1.3 N4 = 9
L4 = 52.0 mm (4 th jump)	2.9	1.9	1.3	



Fig. 7. Feedback mechanism as applied to the small cylinder impinging plane jet.

into the determination of the number of waves, as illustrated in Fig. 7. When both the impinging length used under that velocity and the corresponding cylinder shedding frequency satisfy Eq. (1), that is, the total wave number is an integer, the impinging flow is in the stable condition. In this stable case, the resonant frequency keeps constant and invariant with the impinging length. However, when the wave number does not satisfy the integer condition due to the jet velocity or the impinging length change, for instances, then the resonant frequency will decrease its value until the stable condition is met, that is the wave number becomes another integer. Therefore, the frequency staging or jumping occurs as the function of the exit velocity and the impinging length. Furthermore, as shown in Fig. 6 when the Reynolds number is greater than about 10^4 or $U_0 > 10$ m/s in the present study, the occurrence of the frequency staging remains unchanged. This implies that the flow structure behavior follows very well with the feedback mechanism within this Reynolds number range. However, when the Reynolds number is less than 10⁴, the variation trend of the data staging is still about the same, except their values does not keep constant though. That is because the convective velocity does not keep constant with the jet exit velocity in the calculation of the feedback equation. For more detailed physical explanations about the frequency staging phenomenon due to feedback mechanism, one can refer to our previous papers (Hsiao et al. [3,16]) and Hsu [17].

3.4. Intrinsic feedback mechanism in the small cylinder impinging plane jet

As for the instantaneous flow structures of the small cylinder impinging plane jet at the stable impinging length (L/H = 1.5) (Figs. 8 and 9) and the unstable positions (L/H = 4), similar



Fig. 8. (a) Distributions and (b) constant contour of streamwise mean velocity in the jet-small cylinder impinging flow for $U_0 = 10 \text{ m/s}$ with the cylinder at L/H = 1.5. (Measuring volume does not include the region surrounded by the black box.)

evolution characteristics at typical time are observed as plotted in Fig. 10 at a typical instant of time, which are obtained from the phased-averaging technique explained in Fig. 2. It reveals the parameter that governs the flow dynamics is on the impingement case to be near field or far field, but not on the stable or unstable impinging length. It is like the wave–particle duality suggested by



Fig. 9. (a) Constant contour of dominant frequencies and (b) along $Y_{0.9U_0}$ in streamwise fluctuation velocities for $U_0 = 10$ m/s and L/H = 1.5. (Measuring volume does not include the region surrounded by the black box.)

the physicist, i.e., the small cylinder impinging plane jet can be viewed from two aspects. One is the viewpoint of wave evolution, which comes out as the dynamical properties of the flow field. The other is the perspective on the geometrical set-up in the flow field, which determines the mean flow properties. And the bridge between them is the instability theory developed in the mid-20th century. Based on the former efforts, the argument above can only be a conjecture. In order to identify the deduction above, acoustic excitation at the original free jet fundamental frequency with varicose or sinuous perturbations are applied. Note that the cylinder is also a sound source, the Aeolian tone, the application of the acoustic excitation will have to consider its effect.

In pure impingement condition, the alternating generated resonance wave triggers the initial evolution of the jet shear layers. Therefore, the dominant frequency will be the resonance frequency and the instability wave exhibits sinuous mode in the starting region of evolution in the jet shear layers (see Figs. 11(a) and (b)). In the nearfield impingement, the introduced acoustic excitation intensity is weaker than that of the resonance wave, thus its contribution cannot be distinguished from the phase or the frequency



Fig. 10. Typical phase-averaged vorticity $\langle \omega \rangle$ contours of small cylinder impinging plane jet at (a) L/H = 1.5 and (b) L/H = 4.

measurements of the dominance instability evolution. However, its existence distracts some energy and as a result, the jet shear layers with the introduction of acoustic excitation obtain less amplitude in dominant instability (see Fig. 12). This is reasonable since the instability theory tells that the instability waves may grow in an unstable flow at the beginning, but a specific instability frequency with the largest growth rate will become a dominant one. As for the farfield impingement, the intensity of the applied acoustic wave is larger than the resonance wave, whose amplitudes of the dominance



Fig. 11. Zero crossing cross-correlation distribution between shear layers at various streamwise positions for (a) L/H = 1.5 and (b) L/H = 4, with and without acoustic excitation.

frequency evolution are plotted in Fig. 13. As a consequence, the initial instability waves in the jet shear layers are effectively manipulated by the added acoustic waves regardless of the phase or the frequency variations. As a result, the dynamical behavior of the flow can be identified from the viewpoint of the instability evolution, but not on the mean flow patterns.



Fig. 12. Initial evolution for the integrated amplitudes of the primary instabilities and their subharmonics in jet shear layers at L/H = 1.5.

Concluding the former efforts on the study of the small cylinder impinging plane jet, a modified feedback mechanism model is proposed as demonstrated in Fig. 14. The self-sustained oscillating flow region is like a resonance tube with its ends at the cylinder front and the jet nozzle exit. Two resonance tubes exhibit anti-phase condition on the upper and lower sides near the rear end of the cylinder. The cylinder front can be regarded as the close end of a resonance tube or a node point. Since the feedback pressure wave is generated at the separation points near the rear of the cylinder where the maximum perturbation occurred, the half wavelength can be approximately assumed as the diameter of the cylinder. As for the conventional feedback mechanism in the selfsustained oscillating flow, the convective speed of the feedback wave does not match with that of the evolving instability wave of the shear layers (namely, $U_C = 0.5U_0$ for jet). Owing to the cylinder is purely emerged in the potential core, and no interaction with the coherent structure, the feedback wave travels back from the jet nozzle exit to the cylinder at the speed of the freestream velocity U_0 . Therefore, the number of waves (N = 3 at the first stable stage) calculated by the conventional feedback mechanism represents the number of half-length wave. By the Kelvin–Helmholtz instability, the jet shear layer acts as an amplifier. The dominant instability frequency is selected by the local mean flow properties, and is still followed the subharmonics evolution model. Except the initial region of evolution, especially after the vortex merging, outer influences are hard to affect the jet shear layers.



Fig. 13. Downstream evolution for the integrated amplitudes of the primary instabilities and their subharmonics in jet shear layers at L/H = 4.0.

4. Concluding remarks

The dynamics of the small cylinder impinging plane jet is extensively studied by experimental hot-wire velocity measurements. The small cylinder used for the impingement is small enough to be emerged and placed in the jet center. Current paper summarizes a series of studies from the viewpoint of mean flow properties, dynamics of coherent structures, and the dominant feedback mechanism. The demonstrations of the small cylinder impinging plane jet include the interaction between the cylinder wake and the plane jet, the competition between the jet and wake shear layer instabilities, the standing wave number measured in the self-sustained oscillating flow, and the effect of the application of acoustic excitation. A more intrinsic feedback mechanism is proposed here for the small cylinder impinging plane jet flow, which reveals that the front of the cylinder is a node in the resonance-like tube self-sustained oscillating flow and the diameter of the cylinder is the half wavelength of the resonance wave. The jet shear layer act as a wave amplifier, which can absorb the applied or the induced instability waves (such as the pressure wave of the cylinder or the acoustic excitation. A modified feedback mechanism is also proposed and substantiated successfully by introduction of acoustic waves for excitation in the impinging jet flow.



Fig. 14. Modified feedback mechanism in the small cylinder impinging plane jet.

Appendix A. Nomenclature

sound speed
diameter of the small cylinder
fundamental frequency
acoustic excitation frequency
resonance frequency
wake shedding frequency
height of the plane jet at the exit
feedback constant (= U_c/U_0)
impinging length
1st, 2nd, 3rd frequency jump location
number of waves and an integer in feedback equation
number of waves at 1st, 2nd, 3rd frequency jump location
varicose mode when $m = 0$ and sinuous mode when $m = 1$
correlation function coefficient between u_1 and u_2 or v_1 and v_2
Reynolds number based on jet height
convection speed
mean velocity at the nozzle exit
streamwise velocity fluctuation
amplitude of streamwise velocity fluctuation at specific frequency

v(t)	transverse velocity fluctuation	
v(f)	amplitude of transverse velocity fluctuation at specific frequency	
$\langle w \rangle$	phased average vorticity $\left(= \left\langle \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \right\rangle \right)$	
X, Y	streamwise, transverse position	
$Y_{0.9U_0}$	along the transverse distance where the $U = 0.9 U_0$	
1.5 imp, 4 imp impinging case for $L/H = 1.5$, 4 and no acoustic excitation		
1.5m0, 4m0	impinging case for $L/H = 1.5$, 4 and $m = 0$ acoustic excitation	
1.5m1, 4m1	impinging case for $L/H = 1.5$, 4 and $m = 1$ acoustic excitation	
$ heta_0$	initial momentum thickness of the jet shear layer	

- λ_0 instability wavelength (= $U_0/2f_0$)
- λ_r resonant instability wavelength (= $U_0/2f_r$)
- *v* kinetic viscosity

References

- D. Rockwell, E. Naudascher, Self-sustained oscillations of impinging free shear layers, Annual Review of Fluid Mechanics 11 (1979) 67–94.
- [2] Y.W. Chou, C.K. Cheng, S.S. Sheu, F.B. Hsiao, Self-sustained oscillating vortices of a planar jet impinging upon a small cylinder, C.H. Chen, C. Shih, J. Lienau, R.J. Kung (Eds.), Flow Modeling and Turbulence Measurements VI, *Proceedings of the 6th International Symposium on Flow Modeling and Turbulence Measurements*, A.A. Balkema, Rotterdam, 1996, pp. 133–140.
- [3] F.B. Hsiao, Y.W. Chou, J.M. Huang, The study of self-sustained oscillating plane jet flow impinging upon a small cylinder, *Experiments in Fluids* 27 (1999) 392–399.
- [4] S.C. Crow, F.H. Champagne, Orderly structure in jet turbulence, Journal of Fluid Mechanics 48 (1971) 547-591.
- [5] F.K. Brown, A. Roshko, On density effects and large structure in turbulent mixing layers, Journal of Fluid Mechanics 64 (1974) 775–816.
- [6] C.D. Winant, F.K. Browand, Vortex pairing—the mechanism of turbulent mixing-layer growth at moderate Reynolds number, *Journal of Fluid Mechanics* 63 (1974) 237–255.
- [7] C.M. Ho, N.S. Nosseir, Dynamics of an impinging jet. Part 1. The feedback phenomena, Journal of Fluid Mechanics 105 (1981) 119–142.
- [8] C.M. Ho, F.B. Hsiao, Evolution of coherent structures in a lip jet, Symposium on Structure Complex Turbulent Shear Flow, Marseille, France, 1982, pp. 121–136.
- [9] H. Sato, The stability and transition of a two-dimensional jet, Journal of Fluid Mechanics 7 (1960) 53-80.
- [10] C.M. Ho, E. Gutmark, Preferred modes and the spreading rates of jets, *Physics of Fluids* 26 (1983) 2932–2938.
- [11] V. Strouhal, On one particular way of tone generation, Annals of Physics and Chemistry 5 (1878) 216-251.
- [12] T. von Karman, Collected Works of Theodore von Karman, Vol. 1, Butterworth, London, 1956, pp. 331–338.
- [13] C.H.K. Williamson, Vortex dynamics in the cylinder wake, Annual Review of Fluid Mechanics 28 (1996) 477–539.
- [14] F.B. Hsiao, J.M. Huang, On the evolution of instabilities in the near field of plane jet, *Physics of Fluids A* 2 (1990) 400–412.
- [15] J.M. Huang, F.B. Hsiao, On the mode development in the developing region of a plane jet, *Physics of Fluids* 11 (1999) 1847–1857.
- [16] F.B. Hsiao, I.C. Hsu, J.M. Huang, Smoke-wire visualization and hot-wire measurement of an acoustically excited impinging plane jet with a small cylinder, *Journal of Flow Visualization and Image Processing* 9 (2002) 297–316.
- [17] I.C. Hsu, Instability Mode Behavior of the Acoustically Excited Impinging Plane Jet with a Small Cylinder, Master Thesis, Institute of Aeronautics and Astronautics, National Cheng Kung University, Taiwan, 2002.